CROSS SECTION ESTIMATES FOR dd $\rightarrow \alpha\pi^0$ NEAR THRESHOLD

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In October, 1998, Jerry Miller (U. of Washington) provided two estimates of the isospin-violating cross section for the dd $\rightarrow \alpha\pi^0$ reaction near threshold [Mixx]. These estimates were based on different procedures and led to answers that where within a factor of two of each other. With a sense that neither was correct and wishing to be conservative, the lowest estimate was reduced almost another factor of 2 to $\sigma_{\rm tot} = 100$ pb and included in the Letter of Intent [Ba98]. These original estimates are reviewed and recalculated here, along with some comments on the quality of the input. In view of recent progress on both the analysis and theoretical calculations for the fore-aft cross section asymmetry in np $\rightarrow$ dπ^0 at TRIUMF [Op99,Ni99], I have decided that better estimates should be even lower. In the absence of any more reliable and thorough calculation, these estimates are all that exist as a guide to what can be expected from this experiment.

Each procedure is essentially divided into two parts. The first is a calculation, using straightforward nuclear physics arguments, for the dd $\rightarrow \alpha\pi^0$ cross section assuming that it conserves isospin. The second is an estimate of the ratio of the isospin-violating to the isospin-conserving amplitude.

The Greider method

In 1961, Greider proposed a method to calculate an isospin-conserving dd $\rightarrow \alpha\pi^0$ cross section by scaling from the pd $\rightarrow$ ^{3}Heπ^0 reaction [Gr61]. This involved adding a spectator neutron, and accounting for the various factors of spin statistics, center-of-mass velocity, energy, and wavefunction overlap. His paper was written in response to an experiment to search for the dd $\rightarrow \alpha\pi^0$ reaction using the 460-MeV deuteron beam from the 184-inch Berkeley cyclotron [Po61], and contains an example calculated for that experiment.

The scaling relationship of Greider is

$$\sigma(\mathrm{dd} \rightarrow \alpha\pi^0) = \frac{4}{9}\frac{1}{v_{dd}}\frac{E_4}{E_4+E_\pi}3v_{pd}\frac{E_3+E_\pi}{E_3}\left|\frac{f(\theta)g(\theta)+f(\pi-\theta)g(\pi-\theta)}{2}\right|^2$$

where E_π, E_3, and E_4 are the center-of-mass energies for the pion and the ^{3}He and ^{4}He nuclei and the v's represent velocities. The squared amplitude $|g(\theta)|^2$ is the cross section for pd $\rightarrow$ ^{3}Heπ^0. The sum of the amplitudes antisymmetrizes the expression for the dd system. The remaining overlap is

$$f(\theta) = \int d\mathbf{x}\ \exp(-i\Delta\cdot\mathbf{x})\chi_\alpha(\mathbf{x})\varphi_d(\mathbf{x})$$

where $\chi_\alpha(\mathbf{x})$ and $\varphi_d(\mathbf{x})$ are the normalized single-particle wavefunctions for ^{4}He and the deuteron. The quantity $\Delta = \mathbf{k}/2 - \mathbf{q}/4$ is the momentum transfer for the spectator nucleon with $\mathbf{k}$ and $\mathbf{q}$ being the deuteron and pion momenta. It is tacitly assumed that the momentum of each nucleus is shared equally with its constituent nucleons, thus leading to the factors of 2 and 4. Greider assumed that there are no further normalizations having to do with the ^{3}He wavefunction inside ^{4}He.

For the deuteron, Greider used the Hulthén form

$$\varphi_d(r) = \sqrt{\frac{\pi}{2}} \frac{N}{\xi^2 - \eta^2} \frac{\exp(-\eta r) - \exp(-\xi r)}{r} \ , \quad N^2 = \frac{\eta\xi}{\pi^2}(\eta + \xi)^2$$

with $\eta = 0.232$ fm^{-1} and $\xi = 6\eta$ [Ya54]. The ^{4}He single-particle wavefunction was reported to be

$$\chi_\alpha(x) = \frac{\exp(-\beta x) - \exp(-\gamma x)}{x} + A\frac{\exp(-\beta x) - \exp(-\delta x)}{x} + B\exp(-\delta x)$$

with $\beta = 0.83$, $\gamma = 3.05$, $\delta = 7.25$, and $B = 0.395$ in units of fm^{-1}, and $A = 0.277$. Unlike the Hulthén wavefunction, this one is not normalized. It is too large by a factor of 2.42, and I chose to renormalize by this figure.

This calculation was repeated using the Greider paper as a guide. The overlap integral for $f(\theta)$ was replaced by

$$f(\theta) = 4\pi \int r^2 dr \ j_0(\Delta r)\chi_\alpha(r)\varphi_d(r) \ .$$

Nuclear (as opposed to atomic) masses were used, as well as modern values of all constants. For the sake of definiteness, the calculation was made for pion production at 90° in the center of mass. This matches the 460 MeV experiment and the example in Greider's paper. Near threshold, the momentum transfer is essentially the deuteron momentum (the particles are brought to rest) and is approximately independent of angle. So the scaling factor should apply as well to the total cross section near threshold.

At 460 MeV, the scaling factor between experiments is 0.115, a value that essentially agrees with the result of 0.08 reported by Greider. These estimates are particularly sensitive to the quality of the mass and kinematic input, and there is already a small inconsistency in the kinetic energy of the π^0 at 460 MeV. Jerry apparently ran into the problem of normalization (of the ^{4}He wavefunction?) as well, but he did not include his answer in his notes and I have no information on which to judge what he considers to be good or bad agreement in this case. His assumption was that Greider did it right, but did not put all of the factors that he must have used into the paper. So Jerry calculated the change between 460 and 231.4 MeV, scaled that to Greider's original value, and reported that the scale factor at the lower energy should be 0.16, twice as large. If my calculation is now repeated for 231.4 MeV, the scaling factor grows to 0.269. The ratio is 2.33, a little above Jerry's ratio of 2. At this level, the differences are of no consequence for the evaluation of CE-78.

For the record, most of the change in going from 460 to 231.4 MeV came from the ratio of velocities (1.18 to 1.25) and the overlap $|f(\theta)|^2$ (0.405 to 0.898). The energy factor was close to unity.

One of Greider's major concerns was the quality of the ^{4}He wavefunction. Another way to make an estimate of the ^{4}He single-particle wavefunction is to adjust it to match the ^{4}He charge distribution, now well measured in electron scattering. For a long distance, the charge distribution is essentially Gaussian with a mean radius of $\langle r^2 \rangle^{1/2} = 1.671 \pm 0.014$ fm [Ot85]. The proton charge distribution (single-pole form with $\Lambda = 4.333$) was factored out, and the result transformed to coordinate space. This wavefunction is not so peaked near $r = 0$, giving it a better overlap with the deuteron. The calculated scaling factors are 0.137 for 460 MeV, and 0.308 for 231.4 MeV. The ratio of these two factors is 2.24, close to the ratio found for the wavefunction used by Greider. Thus the scaling in $f(\theta)$ has mostly to do with the size of the spectator nucleon's momentum transfer, which drops from 1.68 to 1.18 fm^{-1} between 460 and 231.4 MeV.

Since it appears that we can reproduce the calculation of Greider by including the normalization of his ^{4}He wavefunction, the best that this method has to offer is given by the present calculations. Thus I have chosen to use the factor of 0.308 between the pd $\rightarrow$ ^{3}Heπ^0 cross section and the "isospin conserving" dd $\rightarrow \alpha\pi^0$ cross section since this incorporates a more modern assessment of the ^{4}He single-particle wavefunction.

Measurements of pd $\rightarrow$ ^{3}Heπ^0 have been reported by Pickar [Pi92]. Rather than extrapolate to $\eta = k_\pi / m_\pi = 0$ to extract the S-wave part alone, we took the simple average of the three most significant data points to be $\sigma/\eta = 12.6$ μb. This overlaps the region ($\eta = 0.2$) where the dd $\rightarrow \alpha\pi^0$ experiment is expected to run.

Estimates of the ratio of isospin-violating to isospin-conserving amplitudes varies considerably. The only serious calculation near threshold has been made by Niskanen for the fore-aft cross section ratio in np $\rightarrow$ dπ^0 [Ni99] since this experiment is underway at TRIUMF. (Niskanen published his calculation after Jerry Miller made his estimates.) This work finds that the isospin violation derives in the main from mixing between the π^0 and η or η' mesons. It is reasonable to assume that the isospin-violating pion production in dd $\rightarrow \alpha\pi^0$ originates in this mechanism, with the neutron and proton coming from different deuterons. Niskanen provides a table which allows us to interpolate the value of the fore-aft asymmetry to be –0.314% at $\eta = 0.2$. A reasonable first estimate for dd $\rightarrow \alpha\pi^0$ is half of this value (obtained when a small cross section change is converted into an amplitude), or –0.157%. At present, this fore-aft asymmetry is less than the systematic errors that now exist in the TRIUMF experimental analysis [about $\pm 0.5\%$, Op99], and this provides an upper bound that is less than a factor of two larger. It is expected that these errors will be reduced by better modelling of the detector response, and that a result that can test the Niskanen prediction will become available. For now, the Niskanen prediction still appears to be an appropriate number that is consistent with what we know about np $\rightarrow$ dπ^0.

This degree of isospin mixing falls at the lower end of the range commonly used in theoretical estimates. It is roughly equal to the fractional neutron-proton mass difference, 0.14%. Other estimates are based on the difference in the current quark masses, $m_d - m_u =$ 3.8 MeV [Mi95], but the problem is to choose the scale of the violation. Dividing by the constituent quark masses of about 1/3 of the nucleon mass gives a mixing of 1.2%, a much larger value. Weinberg [We98] has suggested that the scale should be the pion mass, giving a mixing of 2.8%, but it is not clear where such an enhanced mixing should be observed. The choice of any of these other scales would increase the expected isospin-violating amplitude. In his calculation, Jerry decided to go with the Weinberg estimate of the ratio of amplitudes, thus leading to a cross section much larger than the one we are using here (170 pb [1]). It would appear that all of these larger estimates are inconsistent with the present upper limit on isospin violation in the TRIUMF np $\rightarrow$ ^{3}Heπ^0 reaction.

The factors in the cross section calculation are:

pd $\rightarrow$ ^{3}Heπ^0 cross section of 12.58 η μb
$\eta = k_\pi / m_\pi = 0.202$
scaling factor of 0.308
isospin-violating ratio of 0.00157^2

Combining these factors the dd $\rightarrow$ $\alpha\pi^0$ cross section is expected to be $\sigma_{\text{tot}} = 1.9$ pb at 231.4 MeV.

The "back of the envelope" method

Jerry Miller's first estimate was similar in spirit to what Greider did. But he started with the cross section for the np $\rightarrow$ dπ^0 reaction and added two spectator nucleons. The scaling factor for this process was taken to be the value of a Gaussian that represents the momentum distribution of nucleons in ^{4}He, namely

$$F(p) = \exp\left(-\frac{p^2 R^2}{6}\right)$$

where R is the rms radius of ^{4}He. For the momentum, Jerry chose half of the deuteron center-of-mass momentum (calculated here to be 2.36 fm^{-1}). R was taken from the charge distribution of ^{4}He. This number (1.671) was used earlier in the estimate of a better ^{4}He wavefunction [Ot85]. F is then raised to the fourth power since there are two spectator nucleons and the overlap must be squared when it is applied to a cross section. The scaling I calculate is 0.075, which is somewhat larger than Jerry's value of 0.054.

The cross section for np $\rightarrow$ dπ^0 can be obtained from the S-wave part of the near-threshold cross section reported by Hutcheon [Hu91]. Already at $\eta = 0.2$ there are P-wave contributions, but these should be discarded since odd-L partial waves are excluded from the dd $\rightarrow$ $\alpha\pi^0$ reaction. The value given by Hutcheon is $\sigma/\eta = 92$ μb, scaled down by a factor of 2 to account for the isospin coupling.

At this point Jerry again applied the Weinberg value for estimating an isospin-violating amplitude, and arrived at the answer of 340 pb for the dd $\rightarrow$ $\alpha\pi^0$ total cross section. On the basis described here the factors are

np $\rightarrow$ dπ^0 cross section of 92 η μb
$\eta = 0.202$
scaling factor of 0.075
isospin-violating ratio of 0.00157^2

The product of these factors is $\sigma_{\text{tot}} = 3.4$ pb.

It is actually remarkable that the two estimates come as close as they do. There are clearly many item missing from the "back of the envelope" approach. The wavefunction is again not normalized. The proton charge formfactor has not been removed. There is no accounting for multiple combinations of nucleons that can also lead to pion production. The change in spin statistics is missing. All of these omissions can change things within an order of magnitude, both up and down.

There are also things that both calculations ignore that would be included in some way in a more thorough analysis. Motion inside the nuclei is ignored. Related to this is the observation that a nucleon momentum of half of the deuteron momentum is well below threshold for pion production in the second scheme. One could ask the question a different way by determining what momentum would be required to bring the pion production on-shell and then checking to see what is the probability of finding two nucleons of that momentum within the deuterons. This change tends to reduce the predicted cross section since the deuteron does not contain much strength at the larger momenta.

Beyond this, Niskanen also points out [Ni99] that there are still uncertainties in some of the nuclear physics ingredients in his calculation. One in particular is the strength of the ηNN coupling. Values from boson-exchange fits to NN scattering data still vary by factors of 2 or 3, depending on the treatment of the lighter mesons and the completeness of the development of multiple meson exchange diagrams. Such uncertainties would still remain, even if there were a value for the dd $\to \alpha\pi^0$ cross section and a proper nuclear physics calculation had been done.

Thus these estimates can never be anything more than a rough guide, and planning for CE-78 must consider the contingencies of a cross section either higher or lower than the one given here.

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