# Dominant Rates in CE78: $dd \to \alpha\pi^0$

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**Abstract**

Estimates are made of the two dominant rates anticipated in CE78. Both the systematics of the available data and simple theoretical models lead to an estimate of 120 mb for the angle integrated total cross section for $d(d,p)X$ (the "stripping" cross section). Upon integration over the region of acceptance for CE78, this value drops to 7.5 mb, yielding a count rate of 750 kHz for a luminosity of $10^{32}$. The next most important source of rate, $dd$ elastic scattering, yields a rate almost 20 times smaller.

## 1 Introduction

It is important to document as thoroughly as possible all the considerations leading to our best estimates of the count rates that most influence the design of the detectors to be used in the detection of charged particles in the magnetic channel being constructed for CE78. For the geometry of this experiment, the two dominant contributions to count rate are the $dd \to pX$ breakup reaction, and $dd$ elastic scattering. Other processes generate rates that are orders of magnitude less than these.

## 2 Magnetic Channel Geometry

There are two key features of the geometry of the magnetic channel that must be kept in mind in an estimation of count rates: the bend angle, and the angular acceptance. The angular acceptance can be approximated as a cone of about 2° about the central ray through the channel. The bend angle of principle concern is that given by the 6° magnet in the cooler ring. It will bend the beam deuterons by 6°, and the alphas of interest to CE78 by about 12°. It will also bend protons with the velocity of the beam by about 12°. A very large flux of such protons is expected from the process of deuteron "stripping" by target nuclei in the beam. Substantial contributions to the rate are also anticipated from small angle *dd* elastic scattering. The geometry will accept deuterons from this process in an angular region from approximately $4-8°$ in the lab, with a solid angle of approximately 4 msr.

## 3 *dd* Breakup

The dominant source of rate into the magnetic channel is from the breakup reaction $dd \rightarrow pX$. The major component of this reaction arises from "stripping", which yields a proton with approximately the same rigidity as the alphas sought in $dd \rightarrow \alpha\pi^0$. This process has an enormous cross section at angles near 0°. Although the stopping power of these protons is $\frac{1}{10}$ that of the $\alpha$'s of interest, the rate at which they are produced will determine to a large extent the types of detectors that can be used in the magnetic channel.

### 3.1 Available Data

There are *no* data available for the $dd \rightarrow pX$ and $dd \rightarrow nX$ *inclusive* reactions near zero degrees. There are, however, some data available for the $(d,p)X$ and $(d,n)X$ inclusive reactions. These data span a wide range of energies and (heavier) targets, and allow one to establish some systematics that may be used to estimate cross sections at energies and for targets of concern to CE78.

I tabulate available data in Table 1. It is arranged to yield $\sigma$, the integrated cross section for the 0° region, and $\frac{d\sigma}{d\Omega}$, the differential cross section at 0°. Results obtained using theoretical conversion factors are enclosed in square brackets. The calculation of these factors is straightforward, well supported by the existing experimental data, and discussed in subsection 3.2 (which follows).

The data for carbon are the most comprehensive available. The observed cross sections agree both in size and shape to the predictions of simple theoretical models based on the Serber approach (see subsection 3.2). One principle feature is that $\sigma$, the integrated cross section, is energy independent. Upon examining the systematics, one finds that this appears to be supported by the data. The very early measurements of Schecter *et al.*, [2] appear to be consistently about a factor of 2 too large for each of their targets. The measured integral $\sigma$'s of Jafar *et al.*, [5] are only for angles ranging from $1 - 10^\circ$ in the lab, and are systematically lower than the result for a total integration (as expected). After examining all the results for carbon, one obtains $\sigma = 190 \pm 30$ mb. The shape of the angular distribution is predicted in the Serber approach to be almost entirely determined by the internal momentum distribution of the nucleons in the beam deuteron, and the momentum of the beam deuteron. One can use the model to compute the factor needed to obtain $\frac{d\sigma}{d\Omega}(0^\circ)$ from $\sigma$, or vice-versa. This factor (which depends on beam energy) is 21.0 at $T_d = 270$ MeV, and yields $\frac{d\sigma}{d\Omega}(0^\circ) = 4.0$ b/sr for carbon, in excellent agreement with the unpublished RIKEN results of 4.1 b/sr [3].

Examination of the results for $\sigma$ indicate that for large $A$ we find $\sigma \propto A^{\frac{1}{3}}$, as predicted by the Serber model for cases where the nuclear radius is much larger than the deuteron radius. For smaller $A$, however, we observe $\sigma \propto A^{\frac{2}{3}}$, indicating an increasing importance of the role of nuclear transparency. Assuming such a mass dependence, and a value of 190 mb for C12, we predict a value of 60 mb for a deuteron target. This is a rather large extrapolation. It would perhaps be more correct to scale the cross sections by the ratio of the geometric cross sections. This gives a $\sigma$ for the deuterium target of $(2.2\ \mathrm{fm}/2.8\ \mathrm{fm})^2 \bullet 190\ \mathrm{mb} = 120$ mb.

Finally, we expect the $\sigma$ for deuterium to be less than that for Be. This leads us to an estimate of $\sigma$ for a deuterium target of $120 \pm 60$ mb, based on studies of the available data.

## 3.2 Calculations

The original Serber model [9] does a good job of accounting for the angle and energy distribution of $p$'s and $n$'s obtained in the breakup process $A(d, N)X$ near 0°. It also accounts for more than half of the measured cross section. The other half is accounted for by two simple extensions of the original model, that incorporate the transparency of the nuclear edge and the breakup incurred by the motion of the $d$ through the nuclear field [10]. Each of these extensions yields an energy and angle distribution very similar to that of the original Serber model.

## 3.3 The Serber Model

A key component of the Serber model is that the nucleon detected at small angles comes from the beam deuteron and it has no *direct* interaction with the target nucleus. Thus, its distribution in energy and angle in the laboratory can be explained entirely by the momentum distribution within the deuteron (which has been extremely well determined experimentally and theoretically), and the energy of the incident deuteron. This nucleon is broken free from its associated nucleon, by the direct interaction of that nucleon with the target nucleus. With the exception of high–$Z$ nuclei, this interaction comes almost entirely from the short–ranged strong force [9, 10, 11]. As such, the calculation of the cross section is almost entirely geometric in nature, and becomes much simplified for the case of nuclei with radii much larger than that of the deuteron.

Integrated over ejectile energies, the Serber model yields a prediction of the distribution per unit solid angle that is in good agreement with the data. One obtains [9]:

$$P(\theta)d\Omega \;=\; \frac{1}{2\pi}\,\frac{\theta_0}{[\theta_0^2+\theta^2]^{\frac{3}{2}}}\,d\Omega \tag{1}$$

with the characteristic angle $\theta_0$ that depends on the incident beam energy and is given (in radians) by

$$\theta_0 \;=\; (\epsilon_d/T_d)^{\frac{1}{2}}\,[1-(T_d/8Mc^2)]\,. \tag{2}$$

The quantity $T_d$ (in MeV) is the kinetic energy of the beam deuteron in the lab; $Mc^2$ is 940 MeV; $\epsilon_d$ is 2.18 MeV; and $d\Omega = 2\pi\theta d\theta$ in the small angle approximation. The polar angle at which the yield per unit solid angles has dropped by a factor of 2 from its value at 0° is given by

$$\theta_{\frac{1}{2}} \;=\; 0.7664\,\theta_0 \;. \tag{3}$$

This parameter is tabulated for the available data in Table I.

The distribution in Eqn. 1 can be integrated analytically to obtain the fraction $f$ of the total cross section contained within a cone of angle $\theta_{max}$ about 0° :

$$f \;=\; \int_0^{\theta_{max}} P(\theta) d\Omega \;=\; 1 \;-\; \frac{1}{[1 + (\theta_{max}/\theta_0)^2]^{\frac{1}{2}}} \;. \tag{4}$$

This result is what was used to convert total cross section to differential cross section at 0°, and vice–versa, in Table I. This result will also be needed in our ultimate estimation of count rate, presented in subsection 3.3 .

## 3.4 Monte Carlo Calculation of Cross Section

The success of the Serber approach in computing cross sections for heavy targets [10] motivated a numerical calculation of the breakup cross section at small angles for $d(d,p)X$ using Monte Carlo techniques [12]. To simplify the simulation, an approximate form of the deuteron wave function was used:

$$r\psi(r) \;=\; u(r) \;\cong\; (\frac{\gamma}{2\pi})^{\frac{1}{2}} \,\exp{(-\gamma r)} \,, \tag{5}$$

where $r$ is the distance between the two nucleons, and $1/\gamma = 4.32$ fm. This same approximation was also used in the original calculations of Serber [9]. Squaring this gives us the radial probability distribution function, or the probability for the nucleons to be separated by the distance $r$ :

$$P(r)dr \;=\; \frac{\lambda}{4\pi} \,\exp{(-\lambda r)} \,, \tag{6}$$

where $\lambda \;=\; 2\gamma \;=\; 1/2.16$ fm. The separations of the nucleons in the beam deuteron and in the target deuteron were generated, independently, according to this model distribution. The hard core was approximated by disallowing separations smaller than 0.9 fm. The angles relative to the beam direction were also generated, for each deuteron independently, using Monte

Carlo techniques. A nucleon radius of about 0.9 fm was assumed. If the transverse coordinates of any point of a target nucleon overlapped those of a beam nucleon, those two nucleons were taken to have a strong interaction. A trial was taken to contribute to the cross section if the neutron of the beam deuteron had a strong interaction with either of the nucleons in the target deuteron, while the proton of the beam deuteron had no interaction with either nucleon in the target.

The calculation yielded a value of $97 \pm .5$ mb for the integral cross section, for the case of a nucleon radius of 0.9 fm. Additional calculations with different values for the nucleon radius demonstrated that the computed cross section was directly proportional to the input nucleon radius. Since input values beyond the region of 0.6 – 1.2 fm would be unreasonable, we find that this calculation is in very good agreement with the results obtained by studying the systmatics of data available for heavier targets, that is, a $\sigma$ for $d(d,p)X$ of $120 \pm 60$ mb .

## 3.5 Rate Calculation

At a bombarding energy $T_d = 232$ MeV, about 6 Mev above the threshold for the $dd \rightarrow {}^4He\pi^0$ reaction, one obtains a characteristic angle $\theta_0 = 0.0940$ radians for the breakup distribution, and a half-angle $\theta_{\frac{1}{2}} = 0.0720 = 4.1°$. For a breakup $\sigma = 120$ mb, one obtains a breakup $\frac{d\sigma}{d\Omega}(0°) = 2.16$ b/sr. For a maximum acceptance angle $\theta_{max} = 2.0°$, one will see a fraction $f = 0.0625$ of the total breakup cross section. If one widens the acceptance to $\theta_{max} = 3.0°$, one will see a fraction $f = 0.1264$ of the total breakup cross section.

If we now assume a luminosity $L = 10^{32}$, we obtain a rate $R$ of

$$R \ = \ L \times f \times \sigma = 1.5 \pm 0.7\,\text{MHz}\ , \tag{7}$$

for an acceptance of 3.0°. Decreasing the acceptance to 2.0° reduces the rate by a factor of 2. This calculation clearly indicates that the wire chamber positioned at the entrance to the septum magnet must be capable of handling high rates. The natural choice is a MWPC with a small wire spacing, to minimize the rate per wire.

# 4 *dd* Elastic Scattering

*dd* elastic scattering will be the second most important contribution to the rate at the entrance of the magnetic channel. Unfortunately, there is no information available, either experimental or theoretical, for this cross section at energies that would prove useful in the estimation of rates from this process. One can only make educated guesses as to what it might be, using data available for *pp*, *pn*, and *pd* elastic scattering.

## 4.1 Cross Section

One might guess that an estimate for *dd* elastic scattring could be constructed from cross sections for *pp* and *pn* elastic scattering. This information can be readily obtained in the archival program SAID [13]. Using this program to obtain the differential cross section at a proton bombarding energy of 116 MeV, one finds that the laboratory cross section is fairly constant at 16 mb/sr for lab angles between 6 and 8°, and rises to about 40 mb/sr at 4°. Since the beam deuteron has 2 nucleons, and the target deuteron has 2 nucleons, one might guess that the cross section for *dd* elastic scattering would be 4× that for $NN$ scattering. One could also guess that they could be related by spin degrees of freedom, which would yield a factor of $\frac{9}{2}$, taking into account that one has identical particles in both processes. These considerations lead to a crude estimate of a cross section for *dd* elastic scattering ranging from 60 to 200 mb/sr.

Another estimate might be made by examining data for $d(p,p)d$ elastic scattering (no data for $p(d,d)p$ elastic scattering is available at energies of use to CE78). There recently were published cross sections for $d(p,p)d$ at proton energies ranging from 200 MeV to 300 MeV [14], for laboratory angles ranging from 6 to 15°. At 200 MeV the cross section was about 35 mb/sr at 6°, and dropped to about 15 mb/sr at 15°. After factoring in spin degrees of freedom (a factor of $\frac{3}{2}$), and the different cm-to-lab conversion factors (a factor of 1.67) we might guess the cross section for *dd* elastic to be 40 to 90 mb/sr at $T_d \sim 300$ MeV. The de Broglie wavelength of a 230 MeV deuteron is about 12% larger than that of a 300 MeV deuteron. We would thus expect the cross section to be larger at the lower energy, perhaps in the range of 50 to 150 mb/sr.

We end with the conclusion that the cross section for *dd* elastic scattering will be in the range of 50 to 200 mb/sr.

## 4.2 Rate Calculation

If we assume an average cross section of 100 mb/sr over a cone of 3° (yielding a solid angle of 8.6 msr), we obtain a partial cross section of 0.86 mb, which is 0.057 of the partial cross section for breakup. This will yield a rate of only 86 kHz with a luminosity of $10^{32}$. This is almost 18× less than the rate from breakup.

# 5 Conclusions

The cross sections for the two principle sources of rate in the detectors of the magnetic channel have been estimated. We obtain a differential cross section $\frac{d\sigma}{d\Omega}(0^\circ)$ of $\sim 2$ b/sr for d-breakup, and an elastic cross section in the region of interest of $\sim 100$ mb/sr. Both estimates have an uncertainty of about a factor of 2. Given the anticipated luminosities, neither process will make the experiment unfeasible, ratewise.

However, the experiment does require an absolute determination of cross section, good to better than 30%. The best candidate for a means of monitoring and measuring the luminosity, and its profile near the jet, is *dd* elastic scattering. This means it is essential to obtain a good measurement of this cross section (over angles of use to the experiment). Further, because of the extremely large breakup cross sections (including those not discussed here), any device used to measure and monitor this process must have not only a coincidence, but also the ability to identify protons and deuterons at small angles, and large energies. This will in itself be a challenging task, and must be incorporated into the design of the target box.

# 6 Tables

TABLE I. A listing of measured cross sections for the process of deuteron breakup ($(d,p)X$ and $(d,n)X$), and the sources for these data. A value in square brackets ([]) is a value computed from the associated data in the adjacent column, using the Serber model to obtain the angular distribution.

| $T_d$ (MeV) | $\theta_{\frac{1}{2}}$ (deg) | Particle Detected | Target | $\sigma$ (mb) | $\frac{d\sigma}{d\Omega}(0^\circ)$ (b/sr) | Source |
|---|---|---|---|---|---|---|
| 190 | 4.6 | n | Be, C, Al, Cu, Ag, Pb, U | rel. yields | | [1] |
| 190 | 4.6 | p | C | 350 | [5.18] | [2] |
| | | | U | 2600 | [38.5] | |
| 270 | 3.8 | n | C12 | [195] | 4.1 | [3] |
| | | | Ca40 | [467] | 9.8 | |
| | | | Zr90 | [643] | 13.5 | |
| | | | Pb208 | [1295] | 27.2 | |
| 620 | 2.4 | p | C | 167 | [8.93] | [4] |
| | | | Cu | 301 | [16.1] | |
| 650 | 2.3 | p | C | 119 | 12.2 | [5] (diff. cross section meas. at 1°; integ. sigma for 1 – 10°) |
| | | | Al | 196 | 19.3 | |
| | | | Cu | 296 | 30.7 | |
| | | | Cd | 442 | 52.5 | |
| | | | Pb | 711 | 54.7 | |
| 650 | 2.3 | n | C | [182] | 10.3 | [6] (normalized to [5]) |
| | | | Al | [337] | 19.1 | |
| | | | Cu | [429] | 24.3 | |
| | | | Cd | [664] | 37.6 | |
| | | | Pb | [989] | 56.0 | |
| 800 | 2.0 | n | Be | 172 | [12.5] | [7] |
| | | | C | 205 | [14.9] | |
| | | | Al | 310 | [22.5] | |
| | | | Cu | 469 | [34.1] | |
| | | | Pb | 885 | [64.3] | |
| 1200 | 1.6 | p | Al | 290 | [35.5] | [8] |
| | | | Cu | 550 | [67.3] | |
| | | | Pb | 950 | [116] | |

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